

# Information theory, predictability, and the emergence of complex life

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Despite the obvious advantage of simple life forms capable of fast replication, different levels of cognitive complexity have been achieved by living systems in terms of their potential to cope with environmental uncertainty. Against the inevitable cost associated to detecting environmental cues and responding to them in adaptive ways, we conjecture that the potential for predicting the environment can overcome the expenses associated to maintaining costly, complex structures. We present a minimal formal model grounded in information theory and selection, in which successive generations of agents are mapped into transmitters and receivers of a coded message. Our agents are guessing machines and their capacity to deal with environments of different complexity defines the conditions to sustain more complex agents.

Keywords: Complexity, emergence, computation, evolution, predictability

## I. INTRODUCTION

Simple life forms dominate our biosphere (Gould, 2011) and define a lower bound of embodied, self-replicating systems. But life displays an enormously broad range of complexity levels, affecting many different traits of living entities, from their body size to their cognitive abilities (Bonner, 1988). This creates somewhat a paradox: if larger, more complex organisms are more costly to grow and maintain, why is not all life single-celled? Several arguments help provide a rationale for the emergence and persistence of complex life forms. As an instance (Gould, 2011) proposes that complexity is not a trait explicitly favored by evolution. A review of fossil records convinces Gould that, across genera, phyla, and the whole biosphere, we observe the expected random fluctuations around the more successful adaptation to life. In this big picture, bacteria are the leading life form and the complexity of every other living system is the product of a random drift. Complex life would never be explicitly favored, but a complexity wall exists right below bacteria: simpler forms fail to subsist. Hence, a random fluctuation is more likely to produce more complex forms, falsely suggesting that evolution promotes complexity.

Major innovations in evolution involve the appearance of new types of agents displaying cooperation while limiting conflict (Maynard-Smith & Szathmáry, 1997; Szathmáry & Maynard-Smith, 1997). A specially important innovation involved the rise of cognitive agents, namely those capable of sensing their environments and reacting to their changes in a highly adaptable way (Jablonka & Lamb, 2006). These agents were capable of dealing with more complex, non-genetic forms of information. The advantages of such cognitive complexity become clear when considering their potential to better predict the environment, thus reducing the average hazards of unexpected fluctuations. As pointed by Francois Jacob, an organism is “a sort of machine for predicting the future – an

automatic forecasting apparatus” (Friston, 2013; Jacob, 1998; Wagensberg, 2000). The main message is that *foreseeing* the future is a crucial ability to cope with uncertainty. If the advantages of prediction overcome the problem of maintaining and replicating the costly structures needed for inference, more complex information-processing mechanisms might be favored under the appropriate circumstances.

Here we aim at providing a minimal model that captures these tradeoffs. In doing so, we characterize thoroughly an evolutionary driver that can push towards evermore complex life forms. We adopt an information theory perspective in which agents are inference devices interacting with a Boolean environment. For convenience, this environment is represented by a tape with ones and zeros, akin to non-empty inputs of a Turing machine (figure 1a). The agent  $G$  locates itself in a given position and tries to predict each bit of a given sequence of length  $n$  – hence it is dubbed an  $n$ -guesser. Each attempt to predict a bit involves some cost  $c$ , while a reward  $r$  is received for each successful prediction. 1-guessers are simple and assume that all bits are uncorrelated, while ( $n > 1$ )-guessers find correlations and can get a larger benefit if some structure happens to be present in the environment. A whole  $n$ -bit prediction cycle can be described as a program (figure 1b). A survival function  $\rho$  depends on the number of attempts to guess bits and the number of correct predictions. Successful guessers have a positive balance between reward and prediction cost. They get replicated and pass on their inference abilities. Otherwise, the agent fails to replicate and eventually dies.

As a simple illustration of our approach, consider a 1-guesser living in an infinitely large environment  $E$  where uncorrelated bits take value 0 with probability  $p$  and 1 with probability  $1 - p$ . The average performance of a guesser  $G$  when trying to infer bits from  $E$  is given by

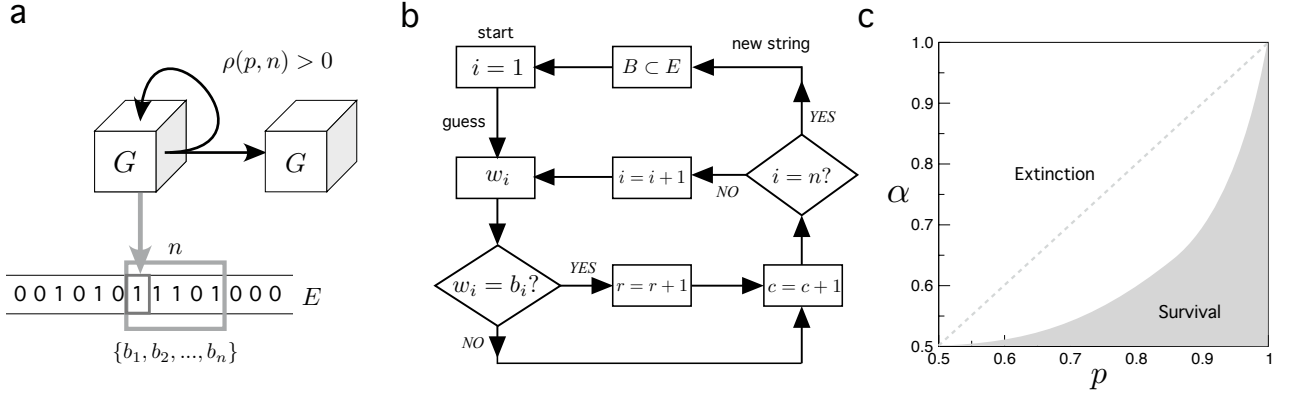


FIG. 1 **Predictive agents and environmental complexity.** **a** An agent  $G$  interacts with an external environment  $E$  that is modeled as a string of random bits. These bits take value 0 with probability  $p$  and value 1 otherwise. The agent tries to guess a sequence of  $n$  bits at some cost, with a reward bestowed for each correctly guessed bit. The persistence and replication of the agent can only be granted if the balance between reward and cost is positive ( $\rho_E^G > 0$ ). **b** For a machine attempting to guess  $n$  bits, an algorithmic description of its behavior is shown as a flow graph. Each loop in the computation involves scanning a random subset of the environment  $B = (b_1, \dots, b_n) \subset E$  by comparing each  $b_i \in B$  to a proposed guess  $w_i$ . **c** A mean field approach to certain kind of 1-guesser (modeled in the text through equations 1, 2, and 3) in environments of infinite size renders a boundary between survival ( $\rho_E^G > 0$ ) and death ( $\rho_E^G < 0$ ) as a function of the cost-reward ratio ( $\alpha$ ) and of relevant parameters for the 1-guesser model ( $p$  in this case). Note that for  $\alpha < 0.5$  every 1-guesser survives for free.

$\bar{p}_E^G$ , the likelihood of emitting a correct guess:

$$\bar{p}_E^G = p^G(0)p + p^G(1)(1-p), \quad (1)$$

where  $p^G(k)$  is the frequency with which the guesser emits the bit value  $k \in \{0, 1\}$ . A strategy that uses  $p^G(0) = p$ ,  $p^G(1) = 1 - p$  (i.e. a guesser that mimics the environment) makes in average

$$\bar{p}_E^G = 2p^2 - 2p + 1 \quad (2)$$

successful predictions. Its survival function reads:

$$\rho_E^G = (2p^2 - 2p + 1)r - c. \quad (3)$$

This curve trivially dictates the average survival or extinction of 1-guessers as a function of the cost-reward ratio  $\alpha \equiv c/r$ . Note that any more complex guesser (like the ones described below) would always fare worst in this case: they would potentially pay a larger cost to infer some structure where none is to be found. Note also that the tunable parameter  $\alpha$  codes for the severity of the environment.

The idea of autonomy and the fact that predicting the future implies performing some sort of computation suggests that a parsimonious theory of life's complexity needs to incorporate reproducing individuals (and eventually populations) and information (they must be capable of predicting future environmental states). These two components define a conflict and an evolutionary trade-off. Being too simple means that the external world is perceived as a source of noise. Unexpected fluctuations can be harmful and useful structure cannot be harnessed in your benefit. Becoming more complex (hence able to infer larger structures, if they exist) implies a risk

of not being able to gather enough energy to support and replicate the mechanisms for inference. As will be shown below, it is possible to derive the critical conditions to survive as a function of the agent's complexity and to connect these conditions to information theory. As advanced above, this allows us to characterize mathematically a scenario in which a guesser's complexity is explicitly selected for.

## II. EVOLUTION AND INFORMATION THEORY

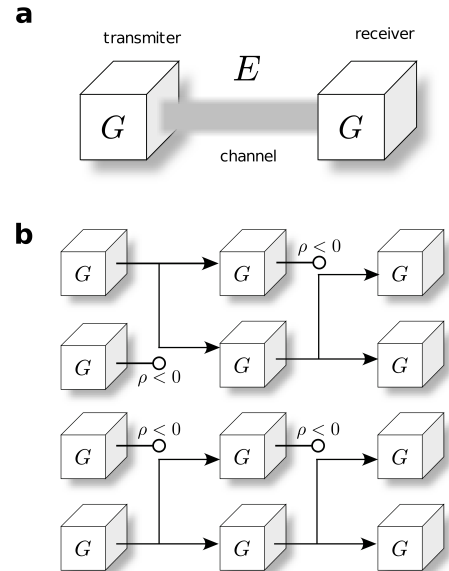
Key aspects of information theory relate deeply to formulations in statistical physics (Jaynes, 1957a,b; Parrondo et al., 2015) and there have been several calls to further integrate information theory in biological research (Joyce, 2002, 2012; Krakauer, 2011; Maynard-Smith, 2000; Nurse, 2008; Walker & Davies, 2012). This theory shall play important roles in population or ecosystems dynamics, in regulatory genomics, and in chemical signal processing among others (Adami, 2012; Bergstrom & Lachmann, 2004; Dall et al., 2005; Dall & Johnstone, 2002; Donaldson-Matasci et al., 2010; Donaldson-Matasci et al., 2008; Evans et al., 2015; Friston, 2013; Hidalgo et al., 2014; Kussell & Leibler, 2005; Marzen & DeDeo, 2016; McNamara & Houston, 1987; Rivoire & Leibler, 2011; Sartori et al., 2014; Segré et al., 2000, 2001; Szathmáry, 1989; Tkačik & Bialek, 2014), but a unifying approach is far from complete. Given its generality and power, information theory has also been used to address problems that connect Darwinian evolution and far from equilibrium thermodynamics (Drossel, 2001; England, 2013; Goldenfeld & Woese, 2010; Nicolis & Prigogine, 1977; Perunov et al., 2014). In its original formulation, Shannon's

information theory (Shannon, 1948; Shannon & Weaver, 1949) considers symbols being conveyed from a transmitter to a receiver through a channel. Shannon only deals with the efficiency of the channel (related to its noise or reliability) and the entropy of the source. This theory ignores the content of the emitted symbols, despite the limitations of such assumption (Corominas-Murtra et al., 2014; Maynard-Smith, 2000).

A satisfactory connection between natural selection and information theory can be obtained by mapping our survival function  $\rho$  into Shannon's transmitter-receiver scheme. To do so we consider replicators at an arbitrary generation  $T$  attempting to "send" a message to (i.e. getting replicated into) a later generation  $T+1$ . A successful "message" thus requires the transmission of information through the survival of the offspring. The former generation acts as a transmitter, the later becomes the receiver, and the environment and its contingencies constitute the channel through which the embodied message must be conveyed (figure 2a). In other words, we can think of a genotype as a generative model (the instructions in an algorithm) that produces the message that must be transmitted. That message would be embodied by a phenotype and it includes every physical process and structure dictated by the generative model. As discussed in (von Neumann & Burks, 1966), any replicating machine must pass on a physically embodied copy of its instructions – hence the phenotype must also include a physical realization of the algorithm encoded by the genotype<sup>1</sup>. Finally, any evolutionary pressure (including the interaction with other replicating signals) can be included as contrivances of the channel.

Following a similar idea of messages being passed from one generation to the next one, (Maynard-Smith, 2000) proposes that the replicated genetic message carries *meaningful information* that must be protected *against* the channel contingencies. Let us instead depart from a replicating message devoid of meaning. We realize that the channel itself would convey more reliably those messages embodied by a phenotype that better deals with the environmental (i.e. channel) conditions. Dysfunctional messages are removed due to natural selection. Efficient signals get more space in successive generations (figure 2b). Through this process *meaningful* bits of environmental information are *pumped* into the replicating signals, such that the information in future messages will *anticipate* those channel contingencies. Meaningful information is not protected against the channel conditions (including noise), but emerges naturally from them.

<sup>1</sup> Note that many of the phenotypic structures build in order to get replicated are later dismissed (think, e.g., about the germ vs. somatic cell lines). We present a clear division between genotype and phenotype for sake of illustration. We are aware of the murky frontier between these concepts.



**FIG. 2 Information and evolution through natural selection.** **a** A simple diagram of the underlying evolution of a population of bit guessers. The survival and replication of a given agent  $G$  is indicated by branching whereas failure to survive is indicated with an empty circle as an endpoint. **b** The propagation of a successful replicator can be understood in terms of a Shannon-like transmission process from one generation to the next.

### A. Messages, channels, and bit guessers

Let us first introduce our implementation of environments (channels), messages, and two classes of agents: transmitters and receivers. Our replicating agents will be dubbed *bit-guessers* because efficient transmission will be equivalent to accurately predicting channel conditions. The notation may seem arid, so it is good to retain a central picture (figure 3): Guessers  $G$  possess a generative model  $\Gamma^G$  that must produce messages that fare well in an environment  $E$ . Both these messages and the environments are modeled as bit strings. What follows is a rigorous mathematical characterization of how the different bit sequences are produced.

We consider  $m$ -environments, strings made up of  $m$  random bits. Bits within  $m$ -environments are sorted, so that it matters what bits follow each other. We might consider one given  $m$ -environment – i.e. one realization ( $E$ ) of  $m$  sorted random bits ( $e_i \in E$ ,  $i = 1, \dots, m$ ,  $e_i \in \{0, 1\}$ ). Alternatively, we might work with the ensemble ( $E_m$ ) of  $m$ -environments – i.e. all possible environments of the same size ( $(e_{i,l} \in E_l) \in E_m$ ,  $l = 1, \dots, 2^m$ ,  $i = 1, \dots, m$ ), or with a sample ( $\hat{E}_m$ ) of this ensemble ( $(E_l \in \hat{E}_m) \subset E_m$ ,  $l = 1, \dots, |\hat{E}_m|$ ). We might evaluate the performance of replicating signals and bit guessers in single  $m$ -environments or in the corresponding ensemble.

We introduce  $m$ -environments to model the conditions of the channel through which messages propagate. To

attempt to transmit an  $n$ -bit message ( $W$ , usually with  $n < m$ ) we extract an  $n$ -sized word ( $B \subset E$ ) from the corresponding environment by picking up the bit at a random position in  $E$  and the successive  $n - 1$  bits. The  $b_i \in B$  are compared to the  $w_i \in W$  (with  $b_i, w_i \in \{0, 1\}$ ) and each  $w_i$  gets through if  $w_i = b_i$ . Hence attempting to get messages through an environment effectively becomes an inference task: if a guesser can anticipate what bit follows, it has a greater chance of sending messages through, and hence moving into a later generation.

In this paper the generative model  $\Gamma^G$  of the guessers is endowed with a minimal ability to react to the environment on the spot (this mechanism will be explained below). Instead of conveying a fixed string,  $\Gamma^G$  builds  $W$  as a function of the broadcast history:

$$w_i = w_i(w_1, \dots, w_{i-1}; b_1, \dots, b_{i-1}).$$

Hence, the evaluation of a generative model is rather based on the ensemble of messages that it can produce. There is a compromise worth investigating between the fidelity of the message that an agent tries to convey and its ability to react to environmental conditions in real time. Exploring this tradeoff is left for future work. The reaction capabilities of our bit-guessers is kept to a minimum.

Usually, a guesser attempts to emit an  $n$ -bit word many ( $N_g$ ) times through the same channel. For each one of these broadcasts, a new  $n$ -sized word  $B^j \subset E$  (with  $b_i^j \in B^j$  for  $j = 1, \dots, N_g$  and  $i = 1, \dots, n$ ) is extracted from the same environment – i.e., as in real life, channel conditions vary. The manufactured message ensemble becomes  $W^j$  with  $w_i^j \in W^j$ .

We can calculate different frequencies with which the guessers or the environments present bits with value  $k, k' \in \{0, 1\}$ :

$$p^G(k; i) = \frac{1}{N_g} \sum_{j=1}^{N_g} \delta(w_i^j, k), \quad (4)$$

$$p_E(k'; i) = \frac{1}{N_g} \sum_{j=1}^{N_g} \delta(b_i^j, k'), \quad (5)$$

$$p_{G,E}(k, k'; i) = \frac{1}{N_g} \sum_{j=1}^{N_g} \delta(w_i^j, k) \delta(b_i^j, k'), \quad (6)$$

$$p_E^G(i) = \frac{1}{N_g} \sum_{j=1}^{N_g} \delta(w_i^j, b_i^j) \Rightarrow \quad (7)$$

$$\Rightarrow \bar{p}_E^G = \frac{1}{n} \sum_{i=1}^n p_E^G(i); \quad (8)$$

with  $\delta(x, y)$  being Dirac's delta. Note that  $p^G(k; i)$  has a subtle dependency on the environment (because  $G$  may react to it) and that  $\bar{p}_E^G$  indicates the average probability that guesser  $G$  successfully transmits a bit through channel  $E$ .

For every bit that attempts to traverse the channel a cost  $c$  is paid. A reward  $r = c/\alpha$  is cashed in only if that bit is successfully received.  $\alpha$  is an external parameter that controls the payoff. The survival function reads:

$$\rho_E^G(\alpha) = (\bar{p}_E^G - \alpha)r. \quad (9)$$

As a rule of thumb, if  $\bar{p}_E^G > \alpha$  the given guesser fares well enough in the proposed environment.

It is useful to quantify the entropy per bit of the messages produced by  $G$ :

$$H(G) = -\frac{1}{n} \sum_{i=1}^n \sum_k p^G(k; i) \log(p^G(k; i)), \quad (10)$$

and the mutual information between the messages and the environment:

$$I(G : E) = \frac{1}{n} \sum_{i=1}^n \sum_{k, k'} p_{G,E}(k, k'; i) \times \log \left( \frac{p_{G,E}(k, k'; i)}{p^G(k; i) p_E(k'; i)} \right). \quad (11)$$

To evaluate the performance of a guesser over an ensemble  $\hat{E}_m$  of environments (instead of over single environments) we attempt  $N_g$  broadcasts over each of  $N_e$  different environments ( $E_l \in \hat{E}_m$ ,  $l = 1, \dots, N_e \equiv |\hat{E}_m|$ ) of a given size. For simplicity, instead of labeling  $b_{i,l}^j$ , we stack together all  $N_g \times N_e$   $n$ -sized words  $W^j$  and  $B^j$ . This way  $b_i^j \in B^j$  and  $w_i^j \in W^j$  for  $i = 1, \dots, n$  and  $j = 1, \dots, N_g N_e$ . We have  $p^G(k; i)$ ,  $p_{\hat{E}_m}(k'; i)$ ,  $p_{G, \hat{E}_m}(k, k'; i)$ ,  $p_{\hat{E}_m}^G(i)$ , and  $\bar{p}_{\hat{E}_m}^G$  defined just as before, only with  $j$  running through  $j = 1, \dots, N_g N_e$ . Also as before, we average the payoff across environments to determine whether a guesser's messages get successfully transmitted or not given the length of the environment ensemble ( $m$ ) and  $\alpha$ :

$$\rho_{\hat{E}_m}^G(\alpha) = (\bar{p}_{\hat{E}_m}^G - \alpha)r. \quad (12)$$

Note that

$$I(G : \hat{E}_m) = \frac{1}{n} \sum_{i=1}^n \sum_{k, k'} p_{G, \hat{E}_m}(k, k'; i) \times \log \left( \frac{p_{G, \hat{E}_m}(k, k'; i)}{p^G(k; i) p_{\hat{E}_m}(k'; i)} \right) \quad (13)$$

is different from

$$\langle I(G : E) \rangle_{\hat{E}_m} = \frac{1}{N_e} \sum_{l=1}^{N_e} I(G : E_l). \quad (14)$$

We use  $\langle \cdot \rangle_{\hat{E}_m}$  to indicate averages across environments of an ensemble  $\hat{E}_m$ .

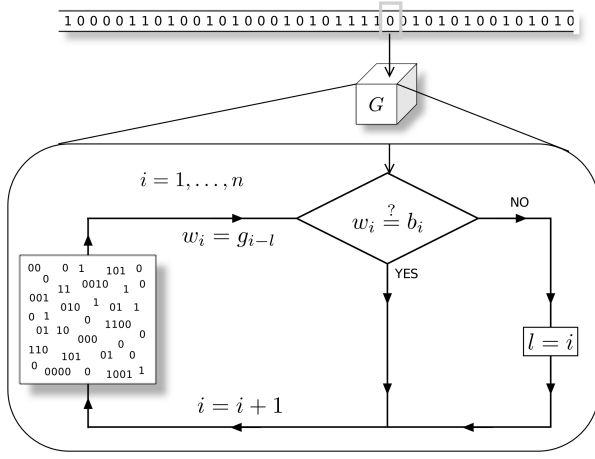


FIG. 3 **From a generative model to inference about the world.** A diagrammatic representation of the algorithmic logic of the bit guessing machine. Our  $n$ -guesser contains a generative model (represented by a pool of words) from which it draws guesses about the environment. If a bit is successfully inferred, the chosen conjecture is pursued further by comparing a new bit. Otherwise, the inference is reset.

The message-generation process  $\Gamma^G$  embodied by our  $n$ -guessers could be implemented in different ways, including Artificial Neural Networks (ANNs) (Hopfield, 1988), spiking neurons (Maass & Bishop, 2001), Bayesian networks (Jensen, 1996; Pearl, 1985), Turing machines (Turing, 1936), Markovian chains (Markov, 1971),  $\epsilon$ -machines (Crutchfield & Young, 1989), Random Boolean Networks (RBNs) (Kauffman, 1993), among others. These devices elaborate their guesses through a series of algorithms (e.g. back-propagation, message passing, or Hebbian learning) provided they have access to a sample of their environment.

In the real world, trial and error and evolution through natural selection would be the algorithm wiring the message generation processes (i.e. the genotype) into our agents. This would establish some dynamics out of which the relevant inference would emerge, which is a very interesting process. However, in this paper we aim at understanding the limits imposed by a channel's complexity and the cost of inference; specially when the performance of the guessers is almost optimal. Therefore, we will assume that our agents perform an almost perfect inference given the environment where they live. This best inference will be hard-wired in the guesser's generative model of their environment  $\Gamma^G$  as explained right ahead.

A guesser's generative model usually depends on the environment where it is deployed, so we note  $\Gamma^G \equiv \Gamma_E^G$ . This  $\Gamma_E^G$  will consist of a pool of bits  $g_i \in \Gamma_E^G$  (figure 3) and a series of rules dictating how to emit those bits: either in a predetermined order or as a response to the channel's changing conditions. Whenever we pick up an environment  $E = \{e_i, i = 1, \dots, m\}$ , the best first guess possible will be the bit (0 or 1) that shows up with more

frequency. Hence:

$$\Gamma_E^G(1) \equiv g_1 = \max_{k'} \{p_E(k'; 1)\}; \quad (15)$$

If both 0 and 1 appear equally often we choose 1 without loss of generality. If the agent succeeds in its first guess, its safest next bet is to emit the bit (0 or 1) that more frequently follows  $g_1$  in the environment. We proceed similarly if the first two bits have been correctly guessed, if the first three bits have been correctly guessed, etc. We define  $p_{B|\Gamma}(k; i)$  as the probability of finding  $k = \{0, 1\}$  at the  $i$ -th position of the  $B^j$  word extracted from the environment, provided that the guess so far is correct:

$$p_{B|\Gamma}(k'; i) = \frac{1}{Z(i)} \sum_{j=1}^m \delta(b_i^j, k') \prod_{i'=1}^{i-1} \delta(b_{i'}^j, g_{i'}). \quad (16)$$

The index  $j$ , in this case, labels all  $n$ -sized words within the environment ( $b_i^j \in B^j \subset E$  and  $Z(i)$  is a normalization constant that depends on how many words in the environment match  $\Gamma_E^G$  up to the  $(i-1)$ th bit:

$$Z(i) = \sum_{j=1}^m \prod_{i'=1}^{i-1} \delta(b_{i'}^j, g_{i'}). \quad (17)$$

It follows:

$$\Gamma_E^G(i = 2, \dots, n) \equiv g_i = \max_{k'} \{p_{B|\Gamma}(k'; i)\}. \quad (18)$$

Note that the pool of bits in  $\Gamma_E^G$  consists of an  $n$ -sized word, which is what they try to emit through (i.e. it constitutes the guess about) the channel. If a guesser would not be able to react to environmental conditions, the word  $W$  that is actually generated at every emission would be the same in every case and  $w_i^j = g_i$  always; but we also allow our guessers a minimal reaction if one of the bits fails to get through the channel (i.e. if one of the guesses is not correct). This minimal reaction capacity by our guessers results in:

$$w_i^j = \Gamma_E^G(i - l) = g_{i-l}, \quad (19)$$

where  $l$  is the largest  $i$  at which  $w_i^j \neq b_i^j$ . This means that a guesser restarts the broadcast of  $\Gamma_E^G$  whenever it makes a mistake<sup>2</sup>.

All together, our guesser consists of a generative model  $\Gamma^G$  that contains a pool of bits and a simple conditional instruction. This is reflected in the flow

<sup>2</sup> Note that more elaborated guessers would not only reset their guess. They might browse through a tree with conditional instructions at every point. Besides an extended memory to store the growing number of branches, they would also require nested *if-else* instructions. On the other hand, ANNs or Bayesian networks might implement such tree-browsing without excessive *if-else* costs.

chart in figure 3. Coming back to biology, and as noted above, the generative model is akin to a genotype and the resulting bits emitted as guesses about the channel would make up a phenotype.

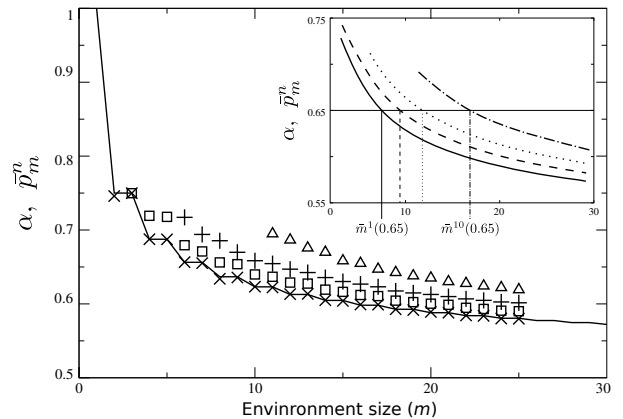
We have made a series of choices regarding how to implement environmental conditions. These choices about implementation affect the way some randomness enters the model (reflected in the fact that, given an environment  $E$ , a guesser might come across different words  $B \subset E$ ) and also how we implement our guessers (including their minimal adaptability to wrong bits). We came up with a scheme that codes guessers, the environment (or channel), and the messages transmitted as bit strings. This allows us a direct measurement of information-theoretical features which are suitable for our discussion, but the conclusions at which we arrive should be general: survival will depend on a replicator's ability to extract useful information from the channel (encoded in the message that it conveys from one generation to the next one), and on the cost-efficiency tradeoff related to *how meaningful* bits are.

Because of the minimal implementation discussed, all guessers of the same size are equal. Environmental ensembles of a given size are considered equivalent as well. Hence, the notation is not affected if we identify guessers and environments by their sizes. Accordingly, in the following we substitute the labels  $G$  and  $E$  by the more informative ones  $n$  and  $m$  respectively. Hence  $\rho_{E_m}^G(\alpha)$  becomes  $\rho_m^n(\alpha)$ ,  $\bar{p}_E^G$  becomes  $\bar{p}_m^n$ , etc.

### III. LIFE COMPLEXITY IS TUNED TO A PREDICTABILITY-REPLICATION TRADEOFF DICTATED BY THE ENVIRONMENT

We report a series of numerical experiments. Some of them deal with guessers in environment ensembles of fixed size, others allow guessers to switch between environment sizes to find a place where to thrive. The important message is that the complexity of the guessers that can populate a given environment (i.e. the complexity of the most successful messages traversing the corresponding channel) is tuned by the predictability of the environment. Environment sizes exist at which simple guessers die off but in which more complex life flourishes – thus offering an explanation to real-life excursions beyond bare replicative success. Our scheme allows us to quantify the different aspects involved in the tradeoff, and to characterize this evolutionary pressure behind complex replicators.

Finally, some of the simulations engage  $n$ -guessers with different  $n$  in direct competition with each other. Yet other experiments explore what may happen if resources could get exhausted. This is suggestive of how the current model will be explored in future research.



**FIG. 4 Probability of correctly guessing a bit in environment ensembles of constant size.**  $\bar{p}_m^n$ , average probability that  $n$ -guessers correctly guess 1 bit in  $m$ -environments for different  $n$  values. Here  $\bar{p}_m^1$  can be computed analytically (solid line in the main plot) and marks an average, lower predictability boundary for all guessers. In the inset, the data has been smoothed and compared to a given value of  $\alpha$  (represented by a horizontal line). At the intersection between this line and  $\bar{p}_m^n$  we find  $\bar{m}^n(\alpha)$ , the environment size at which  $n$ -sized agents guess just enough bits to survive given  $\alpha$ . Notice that  $n$ -guessers are evaluated only in environments of size  $m \geq n$ .

#### A. Guessers isolated in environments of fixed size

Figure 4 shows  $\bar{p}_m^n$ , the average probability that  $n$ -guessers correctly guess 1 bit in  $m$ -environments. The 1-guesser (that lives off maximally decorrelated bits given the environment) establishes a lower bound. More complex machines will guess more bits in average, except for infinite environment size  $m \rightarrow \infty$ , at which point all guessers have equivalent predictive power.

As  $m$  grows, environments (which consist of collections of random bits) get less and less predictable. Importantly, the predictability of shorter words decays faster than that of larger ones, thus enabling guessers with larger  $n$  to survive where others would perish. There are  $2^n$  possible  $n$ -words, of which  $m$  are realized in each  $m$ -environment. When  $m \gg 2^n$ , the environment implements an efficient, ergodic sampling of all  $n$ -words – thus making them maximally unpredictable. When  $n \lesssim m < 2^n$  the sampling of  $n$ -sized words is far from ergodic and a non-trivial structure is induced in the environment because the symmetry between  $n$ -sized words is broken – they cannot be equally represented due to finite size sampling effects.

This allows that complex guessers (those with the ability to contemplate larger words, keep them in memory, and make choices regarding information encoded in larger strings) can guess more bits, in average, than simpler agents. In terms of messages crossing the channel, while shorter words are meaningless and basically get transmitted (i.e. are correctly guessed) by chance alone, larger

words might contain meaningful, non-trivial information that get successfully transmitted because they cope with the environment in the adequate way.

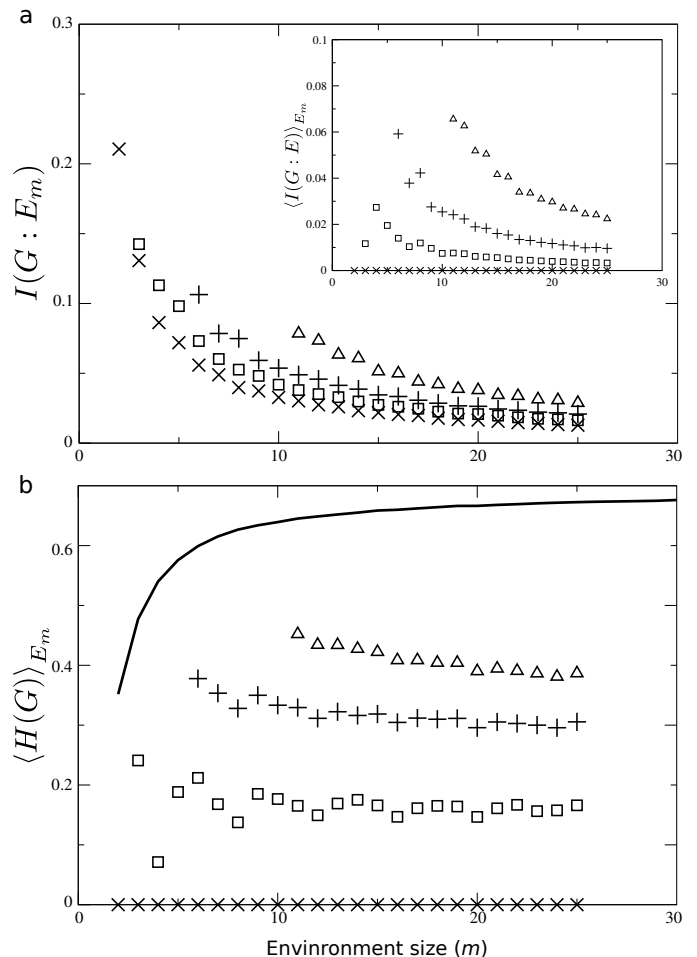
Note that this symmetry breaking to favor predictability of larger words is just a mechanism that allows us to introduce correlations in a controlled and measurable way. In the real world this mechanism might correspond to asymmetries between dynamical systems in temporal or spatial scales. Although our implementation is rather ad hoc (suitable to our computational and conceptual needs), we propose that similar mechanisms might play important roles in shaping life and endowing the universe with meaningful information. Indeed, it might be extremely rare to find a kind of environment in which words of all sizes become non-informative simultaneously.

Back to our experiments, the mutual information between a guesser's response and the environment (i.e. between broadcast messages and channel conditions) further characterizes the advantages of more complex replicators. Figure 5a shows  $I(G : E_m)$  and  $\langle I(G : E) \rangle_{E_m}$ . As we noted above, these quantities are not the same. Let us focus on 1-guesser for a moment to clarify what these quantities encode.

Given an environment, 1-guessers have got just one bit that they try to emit repeatedly. They do not react to the environment – there is not room for any reaction within one bit, so their guess is persistently the same. The mutual information between the emitted bit and the arbitrary words  $B \subset E$  that 1-guessers come across is precisely zero, as shown in the inset of figure 5a. Hence,  $\langle I(G : E) \rangle_{E_m}$  captures the mutual information due to the slight reaction capabilities of guessers to the environmental conditions.

While the bits emitted by 1-guessers do not correlate with  $B \subset E$ , they do correlate with each given  $E$  since they represent the most frequent bit in the environment. Accordingly, the mutual information between a 1-guesser and the aggregated environments (reflected by  $I(G : E_m)$ ) is different from zero (figure 5a). To this quantity contribute both the reaction capability of guessers and the fact that they have hard-wired a near-optimal guess in  $\Gamma_E^G$ , as explained in section II.A.

We take the size of a guesser  $n$  as a crude characterization of its complexity. This is justified because larger guessers can store more complex patterns.  $\langle H(G) \rangle_{E_m}$  indicates that more complex guessers look more entropic than less complex ones (figure 5b). Larger guessers come closer to the entropy level of the environment (black thick line in figure 5b), which itself tends rapidly to  $\log(2)$  per bit. This is remarkable: better performing guessers appear more disordered to an external observer even if they are better predictors when considered within their context. Note that  $\langle H(G) \rangle_{E_m}$  is built based on the bits actually emitted by the guessers, meaning that this quantity correlates with the complexity of the phenotype. For guessers of fixed size  $n$ , we observe a slight decay of  $\langle H(G) \rangle_{E_m}$  as we proceed to larger environments.



**FIG. 5 Mutual information and entropy.** Guessers with  $n = 1$  (crosses),  $n = 2$  (squares),  $n = 5$  (pluses), and  $n = 10$  (triangles) are presented. **a**  $I(G : E_m)$  and  $\langle I(G : E) \rangle_{E_m}$  (inset) quantify the different sources of information that allow more complex guessers to thrive in environments in which simpler life is not possible. **b** The entropy of a guesser's message given its environment seems roughly constant in these experiments despite the growing environment size. This suggests an intrinsic measure of complexity for guessers. Larger guessers look more random even if they might carry more meaningful information about their environment. The thick black line represents the average entropy of the environments (which approaches  $\log(2)$ ) against which the entropy of the guessers can be compared.

## B. Evolutionary drivers

The key question is whether the payoff may be favorable for more complex guessers provided that they need a more costly machinery in order to get successfully reproduced. As discussed above, the cost of units of Artificial Neural Networks or the number of states in Bayesian networks would enter this evolutionary game if such implementations of bit guessers were chosen. To keep the discussion simple, and without loss of generality, bit guessers incur only in a cost proportional to the number of bits



that they try to transmit. Equation 12 captures all the forces involved: the cost of transmitting larger messages versus the reward of a successful transmission that comes after more complex environments could be apprehended.

Guessers of a given size survive in an environment ensemble if, in average, they can guess enough bits of the environment or, put otherwise, if they can convey enough bits through the channel (in any case, they survive if  $\bar{p}_m^n > \alpha$ , which implies  $\rho_m^n > 0$ ). Setting fix a value of  $\alpha$  we find out graphically  $\bar{m}^n(\alpha)$ , the largest environment at which  $n$ -guessers can survive (figure 4, inset). Because  $m$ -environments look more predictable to more complex guessers we have that  $\bar{m}^n(\alpha) > \bar{m}^{n'}(\alpha)$  if  $n > n'$ . This guarantees that for  $\alpha > 0.5$  there always exist  $m$ -environments from which simple life is banned while more complex life can thrive – i.e. situations in which environment complexity is an explicit driver towards more complex life forms. For  $\alpha \leq 0.5$  any guesser survives in average. Under that circumstance a guesser's complexity becomes a neutral trait and we may expect that that complexity does emerge out of sheer fluctuations.

We can simulate some dynamics that gravitate around  $\bar{m}^n(\alpha)$ . After evaluating an  $n$ -guesser  $N_g \cdot N_e$  times in an arbitrary  $m$ -environment, the guesser is promoted to  $m + 1$  if  $\hat{\rho}_m^n(\alpha, N_g, N_e) > 0$ , where  $\hat{\rho}$  represents an empirically accumulated reward instead of the ensemble average. If  $\hat{\rho}_m^n(\alpha, N_g, N_e) < 0$ , the guesser is demoted to  $m - 1$ . The steady state of this process is characterized by a distribution  $P^n(m; \alpha, N_g, N_e) \equiv P^n(m, \alpha)$ , i.e. the frequency with which  $n$ -guessers are found in environments of a given size (figure 6a). The overlap and gaps between  $P^n(m, \alpha)$  for different  $n$  suggest that: i) some guessers would engage in harsh competition if they needed to share environments of a given kind and ii) there is room for different guessers to get segregated into environments of increasing complexity. The average

$$\hat{m}^n(\alpha) = \sum_m m P^n(m, \alpha) \quad (20)$$

should converge to  $\hat{m}^n(\alpha) \simeq \bar{m}^n(\alpha)$  under the appropriate limit – i.e. if we evaluate the guessers numerically enough times as to approach these mean field values. Figure 6b shows dynamically-derived averages  $\hat{m}^n(\alpha)$  and some deviations around them as a function of  $\alpha$ .

It is easily justified that guessers drop to simpler environments if they cannot cope with the complexity. It is less clear why they should seek more complicated environments if they thrive in a given one. This might happen spontaneously if simpler environments get crowded or if resources get exhausted, as we study right ahead.

To simulate a competition dynamics,  $n$ -guessers with  $n = 0, 1, 2, 3$ , and 4 were randomly distributed occupying 100 environments of fixed  $m$  and were assigned an initial  $\hat{\rho}_i(t = 0) = n\rho_0$  with  $i = 1, \dots, 100$  labeling the guesser at each environment. Larger guessers start out with larger  $\hat{\rho}_i(t = 0)$  representing that they come into being with a larger metabolic load satisfied. A 0-guesser represents an unoccupied environment. New empty en-

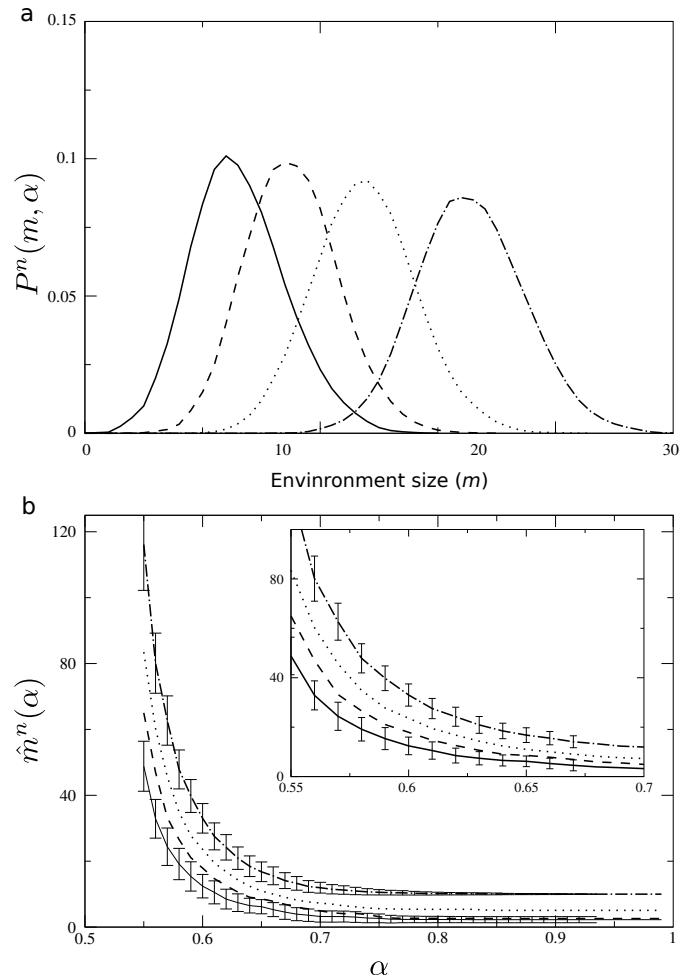
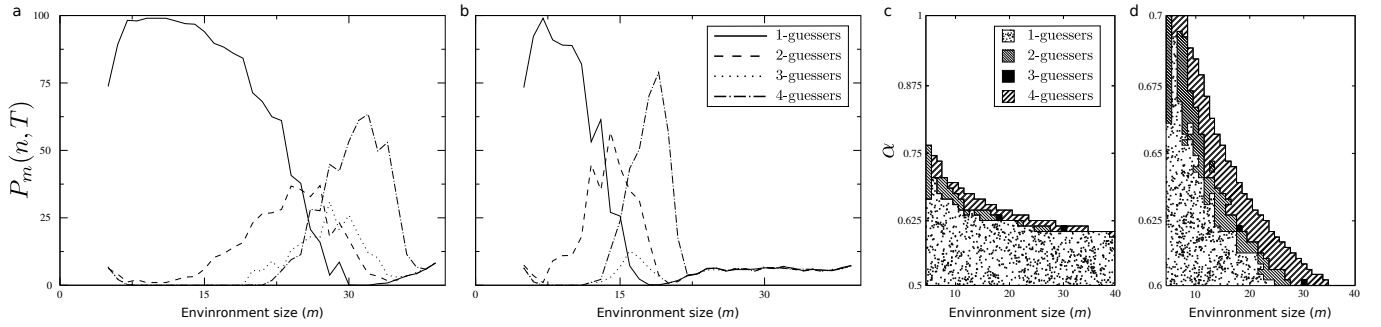


FIG. 6 **Dynamics around  $\bar{m}^n(\alpha)$ .** Again, guessers with  $n = 1$  (solid line),  $n = 2$  (dashed line),  $n = 5$  (dotted line), and  $n = 10$  (dot-dashed line). **a**  $P^n(m, \alpha)$  tells us how often do we find  $n$ -guessers in  $m$ -environments when they are allowed to roam constrained only by their survival function  $\rho_m^n$ . The central value  $\hat{m}^n$  of  $P^n(m, \alpha)$  must converge to  $\bar{m}^n(\alpha)$  and oscillations around it depend (through  $N_g$  and  $N_e$ ) on how often do we evaluate the guessers in each environment. **b** Average  $\hat{m}^n$  for  $n = 1, 2, 5, 10$  and standard deviation of  $P^n(m, \alpha)$  for  $n = 1, 10$ . Deviations are not presented for  $n = 2, 5$  for clarity. The inset represents a zoom in into the main plot.

vironments might appear only if actual ( $n \neq 0$ ) guessers die, as we explain below. We tracked the population using  $P_m(n, t)$ , the proportion of 0-, 1-, 2-, 3-, and 4-guessers through time. (These experiments where the more computationally demanding, that is why we took  $n = 1, 2, 3, 4$  instead of the values  $n = 1, 2, 5, 10$  used throughout the paper. The insights gained from the simulations do not depend on the actual values of  $n$ .)

At each iteration, a guesser (say the  $i$ -th one) was chosen randomly and evaluated with respect to its environment. Then the wasted environment was replaced





**FIG. 7 Evolutionary drivers: competition.** Coexisting replicators will affect each other's environments in non-trivial ways which may often result in competition. We implement a dynamics in which 1-, 2-, 3-, and 4-guessers exclusively occupy a finite number of environments of a given size (fixed  $m$ ). The 100 available slots are randomly occupied at  $t = 0$  and granted to the best replicators as the dynamics proceed. We show  $P_m(n, t = 10000)$  for  $m = 5, \dots, 39$  and  $\alpha = 0.6$  (a),  $\alpha = 0.65$  (b). The most abundant guesser at  $t = 10000$  is shown for  $\alpha \in (0.5, 1)$  (c) and  $\alpha \in (0.6, 0.7)$  (d). Once  $m$  is fixed, there is an upper value of  $\alpha$  above which no guesser survives and all 100 available slots remain empty. Competition and the replication-predictability tradeoff segregate guessers according to the complexity of the environment – i.e. of the transmission channel. Coexistence of different guessers seems possible (e.g.  $m = 15$  in b), but it cannot be guaranteed that the dynamics have converged to a steady distribution.

by a new, random one with the same size. We ensured that every guesser attempts to guess the same amount of bits in average. This means, e.g., that 1-guessers are tested twice as often as 2-guessers, etc. If after the evaluation we found that  $\hat{\rho}_i(t + \Delta t) < 0$ , then the guesser died and it was substituted by a new one. The  $n$  of the new guesser was chosen randomly after the current distribution  $P_m(n, t)$ . If  $\hat{\rho}_i(t + \Delta t) > 2n\rho_0$ , the guesser got replicated and shared its  $\hat{\rho}_i$  with its daughter, who overrode another randomly chosen guesser. This replication at  $2n\rho_0$  represents that parents must satisfy a metabolic load that grows with the size  $n$  of the guessers. There is a range ( $0 < \hat{\rho}_i < 2n\rho_0$ ) within which guessers are alive but do not replicate. Figure 7a and b show  $P_m(n, t = 10000)$  with  $\alpha = 0.6$  and  $0.65$ . Note that for large environments all guessers combined do not add up to 100 – i.e. mostly empty slots remain and most guessers get extinguished. The most abundant guesser after 10 000 iterations is shown in figure 7c as a function of  $m$  and  $\alpha$ .

An alternative evolutionary pressure is introduced if the bits in the environment represent resources that might get exhausted. Thinking from the message broadcasting perspective, a spot on the channel might appear crowded if it is engaged in a successful transmission. Assume that every time that a bit is correctly guessed, it gets exhausted (or gets crowded) with an efficiency  $\beta$  so that in average each bit cannot contribute any reward  $\beta(\bar{p}_m^n/m)$  of the time. The average reward extracted by a guesser from an ensemble (figure 8) becomes:

$$\tilde{r}_m^n = \left(1 - \beta \frac{\bar{p}_m^n}{m}\right) \bar{p}_m^n r. \quad (21)$$

Smaller guessers living in very small environments quickly crowd their channels (alternatively, exhaust the

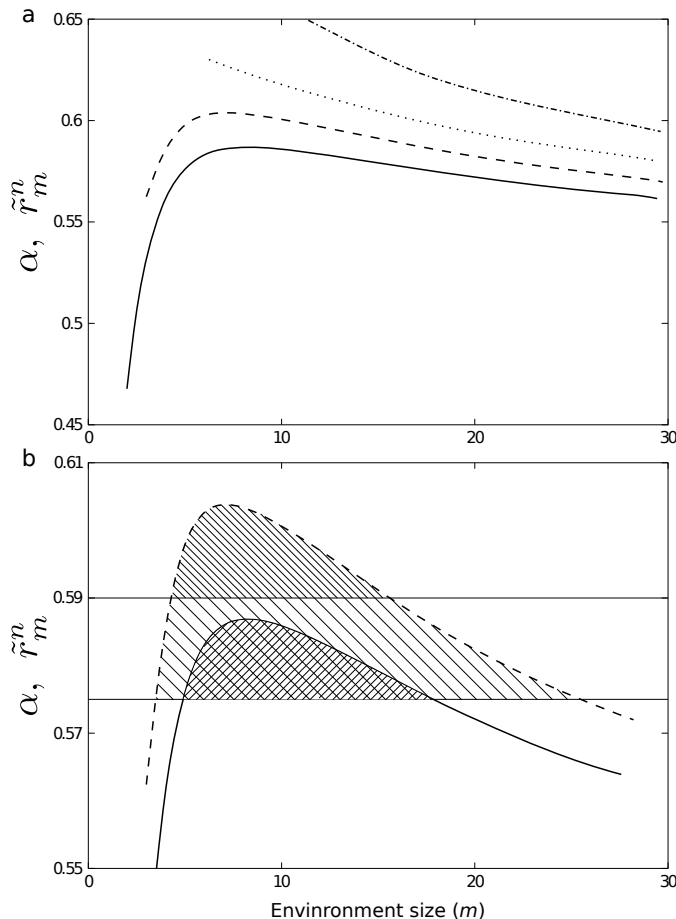
resources they depend on). In figure 8b given some  $\alpha$  and  $\beta = 1$ , 1- and 2-guessers can only survive within some under and upper limits (figure 8b). As announced above, this might establish evolutionary incentives to explore more complex scenarios even if some guesser has its needs already satisfied in a simple environment.

#### IV. DISCUSSION

In this paper we have considered a fundamental question related to the emergence of complexity in living systems. The problem being addressed here is how more complex organisms can overcome the cost of their complexity by developing a higher potential to predict the external environment. As suggested by several authors (Friston, 2013; Jacob, 1998; Wagensberg, 2000) the behavioral plasticity provided by the exploratory behavior of living systems can be understood in terms of their potential for dealing with environmental information (Gerhart and Kirschner, 1997).

Our models make an explicit approach by considering a replication-predictability tradeoff under very general assumptions, namely: (a) More complex environments look more unpredictable to simpler replicators and (b) Agents that can keep a larger memory and make inferences based on more elaborated information can extract enough valuable bits from the environment as to survive in those more challenging situations. Despite the inevitable cost inherent to the cognitive machinery, a selection process towards more complex life is shown to occur. This offers an explicit evolutionary pressure towards more complex life forms.

In our study we identify a transmitter (replicators at a given generation), a receiver (replicators at the next generation), and a channel (any environmental conditions)



**FIG. 8 Evolutionary drivers: exhausted resources.** Rather than monopolizing channel slots (as in figure 5), we can also conceive individual bits as valuable, finite resources that get exhausted whenever they are correctly *guessed*. Then a successful replicator can spoil its own environment and new conditions might apply to where life is possible. **a** Average reward obtained by 1-, 2-, 5-, and 10-guessers in environments of different sizes when bits get exhausted with efficiency  $\beta = 1$  whenever they are correctly guessed. **b** Given  $\alpha = 0.575$  and  $\alpha = 0.59$ , 1- and 2-guessers can survive within upper and lower environment sizes. If the environment is too small, resources get consumed quickly and cannot sustain the replicators. In message transmission language, the guessers crowd their own channel. If the environment is too large, unpredictability takes over for these simple replicators and they perish.

through which a message (ideally instructions about how to build newer replicators) is passed on. Darwinian evolution follows naturally as effective replicators transit a channel faster and more reliably thus getting more and more space in successive generations. The inference task is implicit as the environment itself codes for meaningful bits of information that, if picked up by the replicators, boost the fitness of the phenotypes embodied by the successful messages.

This view is directly inspired by a qualitative ear-

lier picture introduced in (Maynard-Smith, 2000). That metaphor assigned to the DNA some external meaning that had to be preserved *against* the environmental noise. Contrary to this, we propose that, as messages attempt to travel from a generation to the next one, all channel conditions (including noise) pump relevant bits into the transmitted strings – hence there is no need to protect meaning against the channel. The way that we introduce correlations in our scheme (through a symmetry breaking between the information borne by short and larger words due to finite size effects) is compatible with this view. However, interestingly, it also suggests that meaningful information might arise naturally even in highly unstructured environments when different spatial and temporal scales play a relevant role.

This way of integrating information theory and Darwinian evolution is convenient to analyze the questions at hand that concern the emergence of complex life forms. But it also opens further research questions. As discussed at the beginning of the paper, guessers and their transmissible messages might and should shape the transmission channel (e.g., by crowding it, as explored briefly in section III.B). What possible co-evolutionary dynamics between guessers and channels can be established? Are there stable ones, others leading to extinction, etc? Do some of them, perhaps, imply open-ended evolution? Which ones? Also, a guesser's transmitted message might be considered an environment in itself, opening the door to ecosystem modeling based on bare information theory. It is also suggested the exploration of different symbiotic relationships from this perspective and how they might affect coevolution.

Finally, an important question was left aside that concerns the memory vs adaptability tradeoff of bit guessers. Here we studied guessers with a minimal adaptability to focus on the emerging hierarchy of complexity. Adaptability at faster (say, at behavioral) temporal scales is linked to more complex inferences with richer dynamics. This brings in new dilemmas as to how to weight the different building blocks of complex inference – e.g. how do we compare memory and *if-else* or *while* instructions? These and other questions are left for exploration in future research.

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## References

- Adami, C., 2012. The use of information theory in evolutionary biology. *Ann. N. Y. Acad. Sci.*, **1256**(1), pp.49-65.
- Bergstrom, C.T. and Lachmann, M., 2004. Shannon information and biological fitness. In *Information Theory Workshop*, 2004. IEEE (pp.50-54).
- Bonner, J.T., 1988. *The evolution of complexity by means of natural selection*. Princeton University Press.
- Corominas-Murtra, B., Fortuny, J. and Solé, R.V., 2014. Towards a mathematical theory of meaningful communication. *Sci. Rep.*, **4**, 4587.
- Crutchfield, J. P. & Young, K. 1989 Inferring statistical complexity. *Phys. Rev. Lett.*, **63**(2), 105.
- Dall, S.R., Giraldeau, L.A., Olsson, O., McNamara, J.M. and Stephens, D.W., 2005. Information and its use by animals in evolutionary ecology. *Trends Ecol. Evol.*, **20**(4), pp.187-193.
- Dall, S.R. and Johnstone, R.A., 2002. Managing uncertainty: information and insurance under the risk of starvation. *Phil. T. Roy. Soc. B*, **357**(1427), pp.1519-1526.
- Donaldson-Matasci, M. C., Bergstrom, C. T., & Lachmann, M. 2010 The fitness value of information. *Oikos*, **119**(2), 219-230.
- Donaldson-Matasci, M.C., Lachmann, M. and Bergstrom, C.T., 2008. Phenotypic diversity as an adaptation to environmental uncertainty. *Evol. Ecol. Res.*, **10**(4), pp.493-515.
- Drossel, B. 2001 Biological evolution and statistical physics. *Adv. Phys.* **50**(2), 209-295.
- England, J.L., 2013. Statistical physics of self-replication. *The Journal of chemical physics*, **139**(12), p.121923.
- Evans, J.C., Votier, S.C. and Dall, S.R., 2015. Information use in colonial living. *Biol. Rev.*, DOI: 10.1111.
- Friston, K. 2013 Life as we know it. *J. R. Soc. Interface*, **10**(86), 20130475.
- Gerhart, J. and Kirschner, M. 1997. *Cells, embryos, and evolution*. Blackwell Science.
- Goldenfeld, N., & Woese, C. 2010 Life is physics: evolution as a collective phenomenon far from equilibrium. arXiv preprint arXiv:1011.4125.
- Gould, S. J. 2011 *Full house*. Harvard, MA: Harvard University Press.
- Hidalgo, J., Grilli, J., Suweis, S., Muñoz, M.A., Banavar, J.R. and Maritan, A., 2014. Information-based fitness and the emergence of criticality in living systems. *Proc. Nat. Acad. Sci.*, **111**(28), pp.10095-10100.
- Hilbert, M., 2015. Fitness as Informational Fit: The Communication Channel between the Evolving Population and Its Environment. Available at SSRN 2619963.
- Hopfield, J. J. 1988 Artificial neural networks. *IEEE Circuits Devices Mag.*, **4**(5), 3-10.
- Jablonka E and Lamb M.J. 2006. The evolution of information in the major transitions. *J. Theor. Biol.* **239**, 236-246.
- Jacob, F. 1998 *On flies, mice and man*. Harvard, MA: Harvard University Press.
- Jaynes, E.T., 1957. Information theory and statistical mechanics. *Phys. Rev.*, **106**(4), p.620.
- Jaynes, E.T., 1957. Information theory and statistical mechanics. II. *Phys. Rev.*, **108**(2), p.171.
- Jensen, F. V. 1996 *An introduction to Bayesian networks*. London: UCL press.
- Joyce, G. F. 2002 Molecular evolution: Booting up life. *Nature* **420**, 278-279. doi:10.1038/420278a
- Joyce, G. F. 2012 Bit by Bit: The Darwinian Basis of Life. *PLoS Biol.* **10**(5), e1001323. doi:10.1371/journal.pbio.1001323
- Kacian, D. L., Mills, D. R., Kramer, F. R., & Spiegelman, S. 1972 A replicating RNA molecule suitable for a detailed analysis of extracellular evolution and replication. *Proc. Nat. Acad. Sci.*, **69**(10), 3038-3042.
- Kauffman, S. A. 1993 *The origins of order: Self organization and selection in evolution*. Oxford university press.
- Krakauer, D.C., 2011. Darwinian demons, evolutionary complexity, and information maximization. *Chaos*, **21**(3), p.037110.
- Krakauer, D., Bertschinger, N., Olbrich, E., Ay, N. and Flack, J.C., 2014. The information theory of individuality. arXiv preprint arXiv:1412.2447.
- Kussell, E. and Leibler, S., 2005. Phenotypic diversity, population growth, and information in fluctuating environments. *Science*, **309**(5743), pp.2075-2078.
- Maass, W. & Bishop C.M. 2001 *Pulsed neural networks*. Cambridge, MA: MIT Press.
- Markov, A., 1971. Extension of the limit theorems of probability theory to a sum of variables connected in a chain.
- Marzen, S. and DeDeo, S., 2016. Weak universality in sensory tradeoffs. *Physical Review E*, **94**(6), p.060101.
- Maynard-Smith, J. 2000 The concept of information in biology. *Philos. Sci.*, **67**(2), 177-194.
- Maynard-Smith, J. and Szathmáry, E. 1997. *The major transitions in evolution*. Oxford University Press.
- McNamara, J.M. and Houston, A.I., 1987. Memory and the efficient use of information. *J. Theor. Biol.*, **125**(4), pp.385-395.
- von Neumann, J. & Burks, A. W. 1966 Theory of self-reproducing automata. *IEEE Trans. Neural Netw.*, **5**, 3-14.
- Nicolis, G. & Prigogine, I. 1977 *Self-organization in nonequilibrium systems*. New York, NY: Wiley, New York.
- Nurse, P. 2008 Life, logic and information. *Nature* **454**(7203), 424-426.
- Oehlenschläger, F. & Eigen, M. 1997 30 Years Later – a New Approach to Sol Spiegelman's and Leslie Orgel's in vitro EVOLUTIONARY STUDIES Dedicated to Leslie Orgel on the occasion of his 70th birthday. *Origins Life Evol. B.*, **27**(5-6), 437-457.
- Parrondo, J.M., Horowitz, J.M. and Sagawa, T., 2015. Thermodynamics of information. *Nature Phys.*, **11**(2), pp.131-139.
- Pearl, J., 1985. *Bayesian networks: A model of self-activated memory for evidential reasoning*. University of California (Los Angeles). Computer Science Department.
- Perunov, N., Marsland, R. and England, J., 2014. Statistical physics of adaptation. arXiv:1412.1875.
- Rivoire, O. and Leibler, S., 2011. The value of information for populations in varying environments. *J. Stat. Phys.*, **142**(6), pp.1124-1166.
- Sartori, P., Granger, L., Lee, C.F. and Horowitz, J.M., 2014. Thermodynamic costs of information processing in sensory adaptation. *PLoS Comput. Biol.*, **10**(12), p.e1003974.
- Schuster, P. 1996. How does complexity arise in evolution? *Complexity*, **2**(1), 22-30.
- Segré, D., Ben-Eli, D. and Lancet, D., 2000. Compositional genomes: prebiotic information transfer in mutually catalytic noncovalent assemblies. *Proc. Nat. Acad. Sci.*, **97**(8), pp.4112-4117.
- Segré, D., Shenhav, B., Kafri, R. and Lancet, D., 2001. The molecular roots of compositional inheritance. *J. Theor.*

- Biol.*, **213**(3), pp.481-491.
- Shannon, C. E. 2001 A mathematical theory of communication. *Bell Syst. Tech. J.* **27**(3), 379-423. doi:10.1002/j.1538-7305.1948.tb01338.x
- Shannon, C. E. & Weaver, W. 1949 *The Mathematical Theory of Communication*. Univ of Illinois Press, 1949.
- Szathmáry, E., 1989. The integration of the earliest genetic information. *Trends Ecol. Evol.*, **4**(7), pp.200-204.
- Szathmáry, E. and Maynard-Smith, J. 1997. From replicators to reproducers: the first major transitions leading to life. *J. Theor. Biol.* **187**, 555-571.
- Tkačik, G. and Bialek, W., 2014. Information processing in living systems. arXiv preprint arXiv:1412.8752.
- Turing, A. M. 1936 On computable numbers, with an application to the Entscheidungsproblem. *J. of Math* **58**(345-363), 5.
- Wagensberg J. 2000 Complexity versus uncertainty: the question of staying alive. *Biol. Phil.* **15**, 493-508.
- Walker, S. I. & Davies C. W. 2012 The algorithmic origins of life. *J. Phys. Soc. Interface* **10**: 20120869.